Consistent Analysis of the $B \to \pi$ Transition Form Factor in the Whole Physical Region

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Abstract

In the paper, we show that the $B \to \pi$ transition form factor can be calculated by using the different approach in the different q^2 regions and they are consistent with each other in the whole physical region. For the $B \to \pi$ transition form factor in the large recoil regions, one can apply the PQCD approach, where the transverse momentum dependence for both the hard scattering part and the non-perturbative wavefunction, the Sudakov effects and the threshold effects are included to regulate the endpoint singularity and to derive a more reliable PQCD result. Pionic twist-3 contributions are carefully studied with a better endpoint behavior wavefunction for Ψ_p and we find that its contribution is less than the leading twist contribution. Both the two wavefunctions Ψ_B and $\bar{\Psi}_B$ of the B meson can give sizable contributions to the $B \to \pi$ transition form factor and should be kept for a better understanding of the B decays. The present obtained PQCD results can match with both the QCD light-cone sum rule results and the extrapolated lattice QCD results in the large recoil regions.

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I. INTRODUCTION

There are various approaches to calculate the $B \to \pi$ transition form factor, such as the lattice QCD technique[1, 2, 3], the QCD light-cone sum rules (LCSRs)[4, 5, 6, 7] and the perturbative QCD (PQCD) approach[8, 9, 10, 11, 12, 13]. The PQCD calculation is reliable only when the involved energy scale is hard enough, i.e. in the large recoil regions. Due to the restriction to the π energies smaller than the inverse lattice spacing, the lattice QCD calculation becomes more difficult in the large recoil regions and at the present, the lattice QCD results of the $B \to \pi$ transition form factor are available only for soft regions, i.e. $q^2 > 15GeV^2$. The lattice QCD results can be extrapolated to small q^2 regions, and the different extrapolation methods might cause uncertainties about 5%[2]. While, the QCD LCSRs can involve both the hard and the soft contributions below $q^2 < 18GeV^2$[4] and can be extrapolated to higher q^2 regions[5, 6, 7]. Therefore, the results from the PQCD approach, the lattice QCD approach and the QCD LCSRs are complementary to each other, and by combining the results from these three methods, one may obtain a full understanding of the $B \to \pi$ transition form factor in its physical region, $0 \leq q^2 \leq (M_B - M_\pi)^2 \simeq 25GeV^2$.

Certain exclusive process involving hadrons can be described by PQCD if the momentum transfer is sufficiently large. The amplitude can be factorized into the convolution of the non-perturbative wavefunction for each of the hadrons with a PQCD calculable hard-scattering amplitude. The PQCD factorization theorem has been worked out in Refs.[14, 15] based on the earlier works on the applications of PQCD to hard exclusive processes [16]. In the present paper, we shall use the PQCD approach to calculate the $B \to \pi$ transition form factor in the large recoil regions.

In the PQCD approach based on collinear factorization theorem, a direct calculation of the one-gluon-exchange diagram for the B meson transition form factor suffers singularities from the endpoint region of a momentum fraction $x \to 0$. Because of these singularities, it was claimed that $B \to \pi$ transition form factor is dominated by soft dynamics and not calculable in PQCD[17]. In fact, in the endpoint region the parton transverse momenta $\mathbf{k}_\perp$ are not negligible. After including the parton transverse momenta, large double logarithmic corrections $\alpha_s \ln^2 k_\perp$ appear in higher order radiative corrections and must be summed to all orders. In addition, there are also large logarithms $\alpha_s \ln^2 x$ which should also be summed (threshold resummation[18]). The relevant Sudakov form factors from both $k_\perp$ and the

threshold resummation can cure the endpoint singularity which makes the calculation of the hard amplitudes infrared safe, and then the main contribution comes from the perturbative regions.

An important issue for calculating the $B \to \pi$ transition form factor is whether we need to take both the two wavefunctions Ψ_B and $\bar{\Psi}_B$ into consideration or simply Ψ_B is enough? In literature, many authors (see Refs.[9, 10, 11]) did the phenomenological analysis with only Ψ_B, setting $\bar{\Psi}_B = 0$ (or strictly speaking, ignoring the contributions from $\bar{\Psi}_B$). However, As has been argued in Refs.[19, 20], one may observe that the distribution amplitudes (DAs) of those two wavefunctions have a quite different endpoint behavior, such difference may be strongly enhanced by the hard scattering kernel. Even though $\bar{\Psi}_B$ (with the definition in Ref.[13]) is of subleading order contribution, there is no convincing motivation for setting $\bar{\Psi}_B = 0$. In the present paper, we shall keep both the two wavefunctions Ψ_B and $\bar{\Psi}_B$ to do our calculations and show to what extent the $\bar{\Psi}_B$ can affect the final results. Another issue we need to be more careful is about the pionic twist-3 contributions. Based on the asymptotic behavior of the twist-3 DAs, especially $\phi_p^{as}(x) \equiv 1$, most of the people pointed out a large twist-3 contribution[12, 21] to the $B \to \pi$ transition form factor, i.e. bigger than that of the leading twist in almost all of the energy regions. In Ref.[22], the authors have made a detailed analysis on the model dependence of the twist-3 contributions to the pion electro-magnetic form factor, and have raised a new twist-3 wavefunction with a better endpoint behavior for Ψ_p, which is derived from the QCD sum rule moment calculation[23]. And their results show that with such new form for Ψ_p, the twist-3 contributions to the pion electro-magnetic form factor are power suppressed in comparison to the leading twist contributions. According to the power counting rules in Ref.[21], the pionic twist-2 and twist-3 contributions should be of the same order for the case of the B meson decays. With the new form for Ψ_p[22], we show that for the case of the $B \to \pi$ transition form factor, even though the twist-3 contributions are of the same order of the leading twist contributions, its values are less than the leading twist contribution.

The purpose of the paper is to examine the $B \to \pi$ transition form factor in the PQCD approach, and to show how the PQCD results can match with the QCD LCSR results and the extrapolated lattice QCD results. In the PQCD approach, the full transverse momentum dependence (k_T-dependence) for both the hard scattering part and the non-perturbative wavefunction, the Sudakov effects and the threshold effects are included to cure the endpoint

singularity. In section II, based on the k_T factorization formulism, we give the PQCD formulae for the $B \to \pi$ transition form factor in the large recoil regions. In section III, we give our numerical results and carefully study the contributions from Ψ_B and $\bar{\Psi}_B$, and those from the different pionic twist structures. The slope of the obtained form factors $F^{B\pi}_{+,0}(q^2)$ in the large recoil regions can match with those obtained from other approaches. Conclusion and a brief summary are presented in the final section.

II. $B \to \pi$ TRANSITION FORM FACTOR IN THE LARGE RECOIL REGIONS

First, we give our convention on the kinematics. For convenience, all the momenta are described in terms of the light cone (LC) variables. In the LC coordinate, the momentum is described in the form, $k = (\frac{k^+}{\sqrt{2}}, \frac{k^-}{\sqrt{2}}, \mathbf{k}_\perp)$, with $k^\pm = k^0 \pm k^3$ and $\mathbf{k}_\perp = (k^1, k^2)$. The scalar product of two arbitrary vectors A and B is, $A \cdot B = \frac{A^+B^- + A^-B^+}{2} - \mathbf{A}_\perp \cdot \mathbf{B}_\perp$. The pion mass is neglected and its momentum is chosen to be in the minus direction. Under the above convention, we have $P_B = \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_\perp)$, $P_\pi = \frac{M_B}{\sqrt{2}}(0, \eta, \mathbf{0}_\perp)$ and $\bar{P}_\pi = \frac{M_B}{\sqrt{2}}(\eta, 0, \mathbf{0}_\perp)$, with $\eta = 1 - \frac{q^2}{M_B^2}$ and $q = P_B - P_\pi$.

The two $B \to \pi$ transition form factors $F^{B\pi}_+(q^2)$ and $F^{B\pi}_0(q^2)$ are defined as follows:

$$\langle \pi(P_\pi)|\bar{u}\gamma_\mu b|\bar{B}(P_B)\rangle = \left((P_B + P_\pi)_\mu - \frac{M_B^2 - m_\pi^2}{q^2} q_\mu\right) F^{B\pi}_+(q^2) + \frac{M_B^2 - m_\pi^2}{q^2} q_\mu F^{B\pi}_0(q^2), \quad (1)$$

where $F^{B\pi}_+(0)$ should be equal to $F^{B\pi}_0(0)$ so as to cancel the poles at $q^2 = 0$.

The amplitude for the $B \to \pi$ transition form factor can be factorized into the convolution of the wavefunctions for the respective hadrons with the hard-scattering amplitude. The wavefunctions are non-perturbative and universal. The momentum projection for the matrix element of the pion has the following form,

$$M^\pi_{\alpha\beta} = \frac{if_\pi}{4}\left\{ \not{p}\gamma_5\, \Psi_\pi(x, \mathbf{k}_\perp) - m_0^p \gamma_5 \left(\Psi_p(x, \mathbf{k}_\perp) - i\sigma_{\mu\nu}\left(n^\mu \bar{n}^\nu \frac{\Psi'_\sigma(x, \mathbf{k}_\perp)}{6} - p^\mu \frac{\Psi_\sigma(x, \mathbf{k}_\perp)}{6} \frac{\partial}{\partial \mathbf{k}_{\perp\nu}}\right)\right)\right\}_{\alpha\beta}, \quad (2)$$

where f_π is the pion decay constant and m_0^p is the parameter that can be determined by QCD sum rules[23]. $\Psi_\pi(x, \mathbf{k}_\perp)$ is the leading twist (twist-2) wave function, $\Psi_p(x, \mathbf{k}_\perp)$ and $\Psi_\sigma(x, \mathbf{k}_\perp)$ are sub-leading twist (twist-3) wave functions, respectively. $\Psi'_\sigma(x, \mathbf{k}_\perp) = \partial\Psi_\sigma(x, \mathbf{k}_\perp)/\partial x$, $n = (\sqrt{2}, 0, \mathbf{0}_\perp)$ and $\bar{n} = (0, \sqrt{2}, \mathbf{0}_\perp)$ are two null vectors that point to the plus and the minus directions, respectively. The momentum projection for the matrix

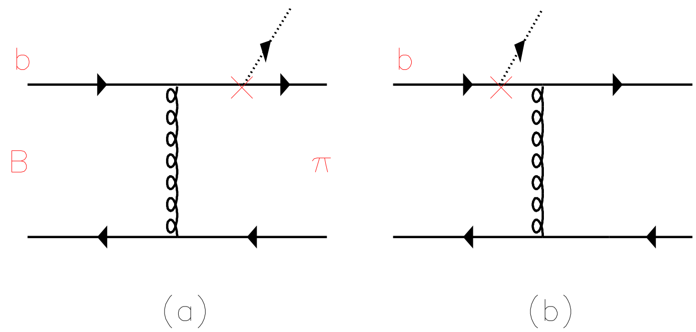


FIG. 1: Lowest order hard-scattering kernel for $B \to \pi$ form factor, where the cross denotes an appropriate gamma matrix.

element of the B meson can be written as [12, 24]:

$$M^B_{\alpha\beta} = -\frac{if_B}{4}\left\{\frac{\not p_B + M_B}{2}\left[\not n\Psi^+_B(\xi,\mathbf{l}_\perp) + \not \bar n\Psi^-_B(\xi,\mathbf{l}_\perp) - \Delta(\xi,\mathbf{l}_\perp)\gamma^\mu\frac{\partial}{\partial l^\mu_\perp}\right]\gamma_5\right\}_{\alpha\beta}, \tag{3}$$

where $\xi = \frac{l^+}{M_B}$ is the momentum fraction for the light spectator quark in the B meson and $\Delta(\xi,\mathbf{l}_\perp) = M_B\int_0^\xi d\xi'(\Psi^-_B(\xi',\mathbf{l}_\perp) - \Psi^+_B(\xi',\mathbf{l}_\perp))$. Note the four-component $l^\mu_\perp$ in Eq.(3) is defined through, $l^\mu_\perp = l^\mu - \frac{(l^+n^\mu + l^-\bar n^\mu)}{2}$ with $l = (\frac{l^+}{\sqrt2}, \frac{l^-}{\sqrt2}, \mathbf{l}_\perp)$.

In the large recoil regions, the $B \to \pi$ transition form factor is dominated by a single gluon exchange in the lowest order as depicted in Fig.(1). In the hard scattering kernel, the transverse momentum in the denominators are retained to regulate the endpoint singularity. The masses of the light quarks and the mass difference ($\bar\Lambda$) between the b quark and the B meson are neglected. The terms proportional to $\mathbf{k}^2_\perp$ or $\mathbf{l}^2_\perp$ in the numerator are dropped, which are power suppressed compared to other $\mathcal{O}(M_B^2)$ terms. Under these treatment, the Sudakov form factor from k_T resummation can be introduced into the PQCD factorization theorem without breaking the gauge invariance[21]. In the transverse configuration b-space and by including the Sudakov form factors and the threshold resummation effects, we obtain the formulae for $F^{B\pi}_+(q^2)$ and $F^{B\pi}_0(q^2)$ as following,

$$\begin{aligned} F^{B\pi}_+(q^2) &= \frac{\pi C_F}{N_c} f_\pi f_B M_B^2 \int d\xi dx \int b_B db_B\ b_\pi db_\pi\ \alpha_s(t) \times \exp(-S(x,\xi,b_\pi,b_B;t)) \\ &\times\ S_t(x)S_t(\xi)\Bigg\{\Bigg[\Psi_\pi(x,b_\pi)\Big((x\eta+1)\Psi_B(\xi,b_B) + (x\eta-1)\bar\Psi_B(\xi,b_B)\Big) \\ &+\ \frac{m_0^p}{M_B}\Psi_p(x,b_\pi)\cdot\left((1-2x)\Psi_B(\xi,b_B) + \left(\frac{2}{\eta}-1\right)\bar\Psi_B(\xi,b_B)\right) - \frac{m_0^p}{M_B}\frac{\Psi'_\sigma(x,b_\pi)}{6}\cdot \end{aligned}$$

$$\left(\left(1+2x-\frac{2}{\eta}\right)\Psi_B(\xi,b_B)-\bar{\Psi}_B(\xi,b_B)\right)+6\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_\pi)}{6}\Psi_B(\xi,b_B)\Bigg]h_1(x,\xi,b_\pi,b_B)$$
$$-\ (1+\eta+x\eta)\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_\pi)}{6}[M_B\Delta(\xi,b_B)]h_2(x,\xi,b_\pi,b_B)$$
$$+\ \Bigg[\Psi_\pi(x,b_\pi)\left(-\xi\bar{\eta}[\Psi_B(\xi,b_B)+\bar{\Psi}_B(\xi,b_B)]+\frac{\Delta(\xi,b_B)}{M_B}\right)+2\frac{m_0^p}{M_B}\Psi_p(x,b_\pi)\cdot$$
$$\left((1-\xi)\Psi_B(\xi,b_B)+(1+\xi-\frac{2\xi}{\eta})\bar{\Psi}_B(\xi,b_B)+2\frac{\Delta(\xi,b_B)}{M_B}\right)\Bigg]h_1(\xi,x,b_B,b_\pi)\Bigg\},\quad (4)$$

and

$$\begin{aligned}
F_0^{B\pi}(q^2) &= \frac{\pi C_F}{N_c}f_\pi f_B M_B^2\int d\xi dx\int b_B db_B\ b_\pi db_\pi\ \alpha_s(t)\times\exp(-S(x,\xi,b_\pi,b_B;t))\\
&\times\ S_t(x)S_t(\xi)\Bigg\{\Bigg[\Psi_\pi(x,b_\pi)\eta\left((x\eta+1)\Psi_B(\xi,b_B)+(x\eta-1)\bar{\Psi}_B(\xi,b_B)\right)\\
&+\ \frac{m_0^p}{M_B}\Psi_p(x,b_\pi)((2-\eta-2x\eta)\Psi_B(\xi,b_B)+\eta\bar{\Psi}_B(\xi,b_B))\\
&-\ \frac{m_0^p}{M_B}\frac{\Psi'_\sigma(x,b_\pi)}{6}\cdot(\eta(2x-1)\Psi_B(\xi,b_B)-(2-\eta)\bar{\Psi}_B(\xi,b_B))\\
&+\ 6\frac{m_0^p}{M_B}\eta\frac{\Psi_\sigma(x,b_\pi)}{6}\Psi_B(\xi,b_B)\Bigg]h_1(x,\xi,b_\pi,b_B)\\
&-\ [3-\eta-x\eta]\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_\pi)}{6}[M_B\Delta(\xi,b_B)]h_2(x,\xi,b_\pi,b_B)\\
&+\ \Bigg[\Psi_\pi(x,b_\pi)\eta\left(\xi\bar{\eta}(\Psi_B(\xi,b_B)+\bar{\Psi}_B(\xi,b_B))+\frac{\Delta(\xi,b_B)}{M_B}\right)\\
&+\ 2\frac{m_0^p}{M_B}\Psi_p(x,b_\pi)\cdot\Big((\eta(1+\xi)-2\xi)\Psi_B(\xi,b_B)+\eta(1-\xi)\bar{\Psi}_B(\xi,b_B)\\
&+\ 2(2-\eta)\frac{\Delta(\xi,b_B)}{M_B}\Big)\Bigg]h_1(\xi,x,b_B,b_\pi)\Bigg\},
\end{aligned}\tag{5}$$

where

$$\begin{aligned}
h_1(x,\xi,b_\pi,b_B) &= K_0(\sqrt{\xi x\eta}\ M_B b_B)\Big[\theta(b_B-b_\pi)I_0(\sqrt{x\eta}\ M_B b_\pi)K_0(\sqrt{x\eta}\ M_B b_B)\\
&\quad+\theta(b_\pi-b_B)I_0(\sqrt{x\eta}\ M_B b_B)K_0(\sqrt{x\eta}\ M_B b_\pi)\Big],
\end{aligned}\tag{6}$$

$$\begin{aligned}
h_2(x,\xi,b_\pi,b_B) &= \frac{b_B}{2\sqrt{\xi xy}M_B}K_1(\sqrt{\xi x\eta}\ M_B b_B)\Big[\theta(b_B-b_\pi)I_0(\sqrt{x\eta}\ M_B b_\pi)K_0(\sqrt{x\eta}\ M_B b_B)\\
&\quad+\theta(b_\pi-b_B)I_0(\sqrt{x\eta}\ M_B b_B)K_0(\sqrt{x\eta}\ M_B b_\pi)\Big],
\end{aligned}\tag{7}$$

and we have set,

$$\Psi_B=\frac{\Psi_B^+ + \Psi_B^-}{2}\ ,\quad \bar{\Psi}_B=\frac{\Psi_B^+ - \Psi_B^-}{2}\ .\tag{8}$$

The functions I_i (K_i) are the modified Bessel functions of the first (second) kind with the i-th order. The angular integrations in the transverse plane have been performed. The factor $\exp(-S(x,\xi,b_\pi,b_B;t))$ contains the Sudakov logarithmic corrections and the renormalization group evolution effects of both the wave functions and the hard scattering amplitude,

$$S(x,\xi,b_\pi,b_B;t) = \left[s(x,b_\pi,M_b) + s(\bar{x},b_\pi,M_b) + s(\xi,b_B,M_b) - \frac{1}{\beta_1}\ln\frac{\hat{t}}{\hat{b}_\pi} - \frac{1}{\beta_1}\ln\frac{\hat{t}}{\hat{b}_B}\right], \quad (9)$$

where $\hat{t} = \ln(t/\Lambda_{QCD})$, $\hat{b}_B = \ln(1/b_B\Lambda_{QCD})$, $\hat{b}_\pi = \ln(1/b_\pi\Lambda_{QCD})$ and $s(x,b,Q)$ is the Sudakov exponent factor, whose explicit form up to next-to-leading log approximation can be found in Ref.[15]. $S_t(x)$ and $S_t(\xi)$ come from the threshold resummation effects and here we take a simple parametrization proposed in Refs.[21, 25],

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c , \quad (10)$$

where the parameter c is determined around 0.3 for the present case.

The hard scale t in $\alpha_s(t)$ and the Sudakov form factor might be varied for the different hard scattering parts and here we need two t_i[13, 21], whose values are chose as the largest scale of the virtualities of internal particles, i.e.

$$t_1 = \mathrm{MAX}\left(\sqrt{x\eta}M_B, 1/b_\pi, 1/b_B\right), \; t_2 = \mathrm{MAX}\left(\sqrt{\xi\eta}M_B, 1/b_\pi, 1/b_B\right). \quad (11)$$

The Fourier transformation for the transverse part of the wave function is defined as

$$\Psi(x,\mathbf{b}) = \int_{|\mathbf{k}|<1/b} d^2\mathbf{k}_\perp \exp\left(-i\mathbf{k}_\perp\cdot\mathbf{b}\right)\Psi(x,\mathbf{k}_\perp), \quad (12)$$

where Ψ stands for Ψ_π, Ψ_p, Ψ_σ, Ψ_B, $\bar{\Psi}_B$ and Δ, respectively. The upper edge of the integration $|\mathbf{k}_\perp| < 1/b$ is necessary to ensure that the wave function is soft enough[26].

In summary, we compare the results in Eqs.(4,5) with those in Refs.[12, 13, 20, 21]. In Ref.[20], only leading twist (Ψ_π) of the pion is discussed. Setting the twist-3 terms to zero, the two formulae in Eqs.(4,5) and Ref.[20] are in agreement. In Ref.[21], the single B meson wave function Ψ_B is assumed and the terms of $\bar{\Psi}_B$ and Δ are neglected. And in Ref.[13], with a new definition for Ψ_B and $\bar{\Psi}_B$, i.e.

$$\Psi_B = \Psi_B^+ , \quad \bar{\Psi}_B = (\Psi_B^+ - \Psi_B^-), \quad (13)$$

both contributions from Ψ_B and $\bar{\Psi}_B$ are taken into consideration, with only the terms of Δ are neglected. The momentum projector used in [13, 21] for the pion is different from the

present projector in Eq.(2), i.e. there is no term proportional to Ψ_σ in Refs.[13, 21]. Except for these differences[1], the formulae in [13, 21] are consistent with ours. Our results agree with Ref.[12], except for several minus errors that should be corrected there.

III. NUMERICAL CALCULATIONS

In the numerical calculations, we use

$$\Lambda^{(n_f=4)}_{\overline{MS}} = 250MeV, \quad f_\pi = 131MeV, \quad f_B = 190MeV, \quad m_0^p = 1.30GeV. \tag{14}$$

The wavefunctions in the compact parameter b-space, $\Psi_+^B(\xi, b_B)$, $\Psi_-^B(\xi, b_B)$, $\Psi_\pi(x, b_\pi)$, $\Psi_p(x, b_\pi)$ and $\Psi_\sigma(x, b_\pi)$ can be found in the appendix. The k_T-dependence has been kept in both the B meson and the pion wavefunctions. As has been argued in several papers[22, 27, 28, 29], the intrinsic k_T-dependence of the wave function is important and the results will be overestimated without including this effect, so it is necessary to include the transverse momentum dependence into the wave functions not only for the B meson but also for the pion. As has been argued in Ref.[22], we take $m_0^p = 1.30GeV$ for latter discussions, which is a little below the value given by the chiral perturbation theory[30].

The two wavefunctions Ψ_B and $\bar{\Psi}_B$ of the B meson shown in the appendix depend only on the effective mass ($\bar{\Lambda} = M_B - m_b$) of the B meson. An estimate of $\bar{\Lambda}$ using QCD sum rule approach gives $\bar{\Lambda} = 0.57 \pm 0.07GeV$[31]. In Fig.(2), we show the $B \to \pi$ transition form factor with different value of $\bar{\Lambda}$, where the shaded band is drawn with a broader range for $\bar{\Lambda}$, i.e. $\bar{\Lambda} \in (0.4GeV, 0.7GeV)$. And for comparison, we show the QCD LCSR result [5] in solid line and its theoretical error ($\pm 10\%$) by a fuscous shaded band in Fig.(2). The results show that the $B \to \pi$ transition form factor will decrease with the increment of $\bar{\Lambda}$. When $\bar{\Lambda} \in (0.5GeV, 0.6GeV)$, one may observe that the present results agree well with the QCD LCSR results[4, 5] up to $q^2 \sim 14GeV^2$. In Fig.(2), for simplicity, only the QCD LCSR results of Ref.[5] are shown. The LCSR results in Refs.[4, 5] are in agreement with each other even though they have taken different ways to improve the QCD LCSR calculation precision, i.e. in Ref.[4], an alternative way to do the QCD LCSR calculation is adopted in which the pionic twist-3 contributions are avoided by calculating the correlator with a proper chiral

[1] According to the power counting rules in Ref.[21], the terms that do not existent in Ref.[21] are defined as sub-leading terms in $1/M_B$ and are neglected accordingly. And here, we keep all the terms with care.

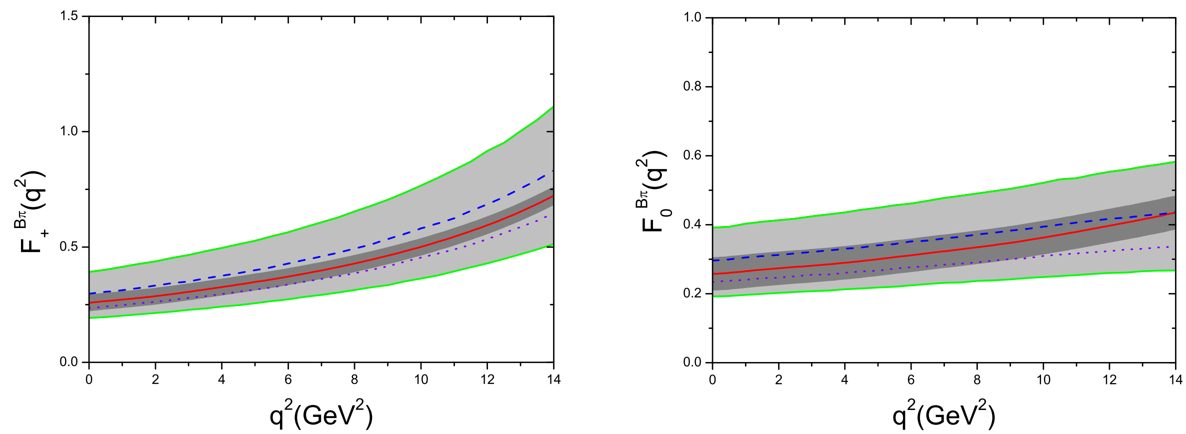
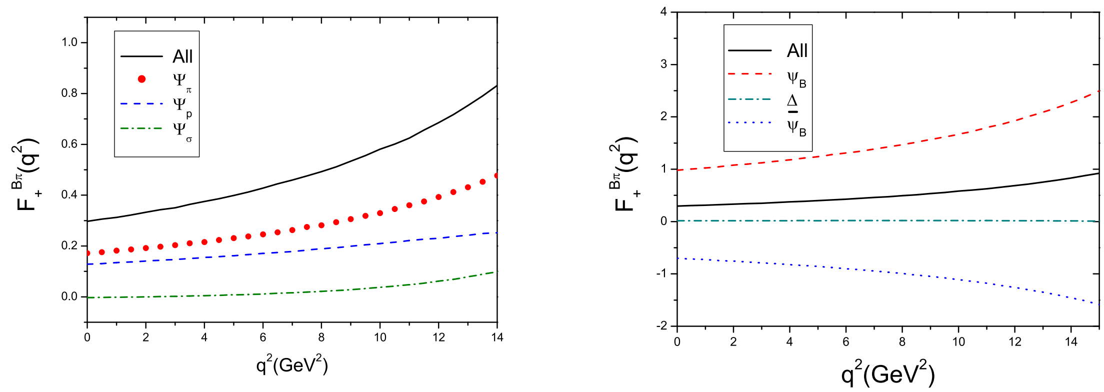


FIG. 2: PQCD results for the $B \to \pi$ transition form factors $F_+^{B\pi}(q^2)$ (Left) and $F_0^{B\pi}(q^2)$ (Right) with different values for $\bar{\Lambda}$. The dashed line stands for $\bar{\Lambda} = 0.5GeV$, the dotted line stands for $\bar{\Lambda} = 0.6GeV$, the upper edge of the shaded band corresponds to $\bar{\Lambda} = 0.40GeV$ and the lower edge of the band corresponds to $\bar{\Lambda} = 0.70GeV$. For comparison, the solid line comes from the QCD LCSR[4, 5] and the fuscous shaded band shows its theoretical error $\pm 10\%$.

current and then the leading twist contributions are calculated up to next-to-leading order; while in Ref.[5], the usual QCD LCSR approach is adopted and both the twist-2 and twist-3 contributions are calculated up to next-to-leading order. In Ref.[13], $\bar{\Lambda}$ is treated as a free parameter and a bigger value is adopted there, i.e. $\bar{\Lambda} = (0.70 \pm 0.05)GeV$. The main reason is that in the present paper, we have used an improved form (with better endpoint behavior than that of the asymptotic one) for the pionic twist-3 wavefunction Ψ_p, while in Ref.[13], they took ϕ_p in Ref.[7] (with an endpoint behavior even worse than the asymptotic one) other than Ψ_p to do the calculations, so the value of $\bar{\Lambda}$ in Ref.[13] must be big enough to suppress the endpoint singularity coming from the hard kernel. For clarity, if not specially stated, we shall fix $\bar{\Lambda}$ to be $0.5GeV$ in the following discussions.

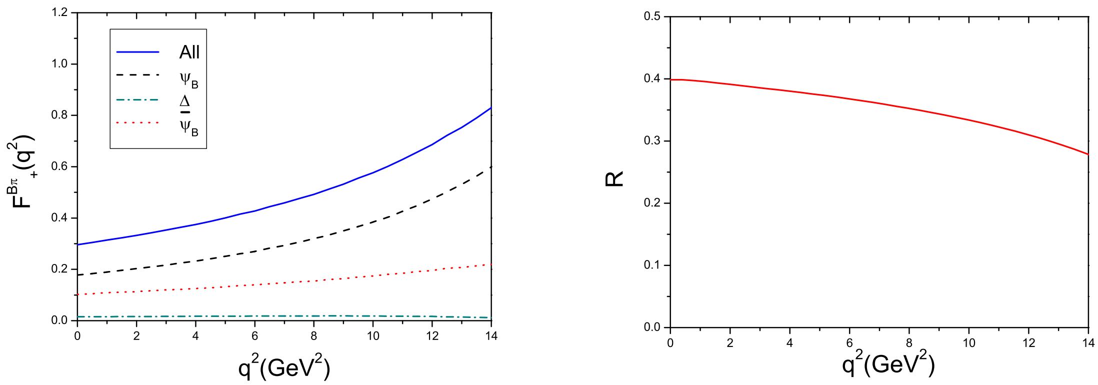
Second, to get a deep understanding of the $B \to \pi$ transition form factor, we discuss the contributions from different parts of the B meson wavefunction or the pion wave function, correspondingly. Here we take $F_+^{B\pi}(q^2)$ to do our discussions and the case of $F_0^{B\pi}(q^2)$ can be done in a similar way. In Fig.(3a), we show the contributions from the different twist structures of the pion wave function, i.e. Ψ_π, Ψ_p and Ψ_σ (the contributions from the terms involving Ψ'_σ are included in Ψ_σ), respectively. From Fig.(3a), one may observe that the



(a) (b)

FIG. 3: PQCD results for the $B \to \pi$ transition form factor $F_+^{B\pi}(q^2)$ with fixed $\bar{\Lambda} = 0.5GeV$. The left diagram is for the different pion twist structures, Ψ_π, Ψ_p and Ψ_σ. The right diagram is for the different B meson structures, Ψ_B, $\bar{\Psi}_B$ and Δ, where Ψ_B and $\bar{\Psi}_B$ are defined in Eq.(8).

contribution from Ψ_π is the biggest, then comes that of Ψ_p and Ψ_σ. And the ratio between all the twist-3 contributions and the leading twist contribution is $\sim 70\%$ in the large recoil regions. This behavior is quite different from the conclusion that has been drawn in Refs.[12, 21], in which they concluded that the twist-3 contribution is bigger than that of twist-2 contribution, especially in Ref.[12], it claimed that the twist-3 contribution is about three times bigger than that of twist-2 at $q^2 = 0$. Such kind of big twist-3 contributions are due to the fact that they only took the pion distribution amplitudes into consideration (or simply adding a harmonic transverse momentum dependence for the pion wavefunctions), and then the endpoint singularity coming from the hard kernel can not be effectively suppressed, especially for Ψ_p whose DA's asymptotic behavior is $\phi_p \equiv 1$. In Ref.[22], the authors have made a detailed analysis on the model dependence of the twist-3 contributions to the pion electro-magnetic form factor, and have raised a new twist-3 wavefunction (as is shown in the appendix) with a better endpoint behavior for Ψ_p, which is inspired from QCD sum rule moment calculation. With this model wave function for Ψ_p, Ref.[22] shows that the twist-3 contributions of the pion electro-magnetic form factor agree well with the power counting rule, i.e. the twist-3 contribution drops fast and it becomes less than the twist-2 contribution at $Q^2 \sim 10GeV^2$. For the present B meson case, according to the power counting rules in Ref.[21], the twist-3 contribution and the twist-2 contribution are of the same order, however



(a) (b)

FIG. 4: PQCD results for the $B \to \pi$ form factor $F_+^{B\pi}(q^2)$ with fixed $\bar{\Lambda} = 0.5GeV$, where Ψ_B and $\bar{\Psi}_B$ are defined in Eq.(13). The left diagram shows the contributions from different B meson wavefunctions, Ψ_B, $\bar{\Psi}_B$ and Δ, respectively. The right diagram is the distribution of the ratio $R = \left(\frac{F_+^{B\pi}|_{\bar{\Psi}_B}}{F_+^{B\pi}|_{All}}\right)$ versus q^2.

one may find from Fig.(3a) that with a new form with better endpoint behavior for Ψ_p, the twist-3 contribution can be effectively suppressed and then its contribution is less than the leading twist contribution.

Now, we show to what extent, $\bar{\Psi}_B$ will affect the final results. Fig.(3b) presents the contributions from Ψ_B, $\bar{\Psi}_B$ and Δ respectively, where Ψ_B and $\bar{\Psi}_B$ are defined in Eq.(8). From Fig.(3b), one may observe that the contribution from Δ is quite small and can be safely neglected as has been done in most of the calculations. However the contribution from $\bar{\Psi}_B$ is quite large, i.e. at $q^2 = 0$, the ratio between the contributions of $\bar{\Psi}_B$ and Ψ_B is about (-70%), which roughly agrees with the observation in Ref.[12]. So the negative contribution from $\bar{\Psi}_B$ can not be neglected, and it is necessary to suppress the big positive contribution from Ψ_B so as to get a more reasonable total contributions from both Ψ_B and $\bar{\Psi}_B$. The above results of Fig.(3b) is obtained by using the definition Eq.(8). A new definition (13) for Ψ_B and $\bar{\Psi}_B$ has been raised in Ref.[13] and the contributions from the Ψ_B, $\bar{\Psi}_B$ and Δ with such a new definition (13) are shown in Fig.(4a). We draw the distribution of the corresponding ratio $R = \left(\frac{F_+^{B\pi}|_{\bar{\Psi}_B}}{F_+^{B\pi}|_{All}}\right)$ versus q^2 in Fig.(4b), where $\left(F_+^{B\pi}|_{\bar{\Psi}_B}\right)$ means that only the contributions from $\bar{\Psi}_B$ are considered and $\left(F_+^{B\pi}|_{All}\right)$ means that all the contributions from the B meson wavefunctions are taken into consideration. One may observe from Fig.(4b) that even with

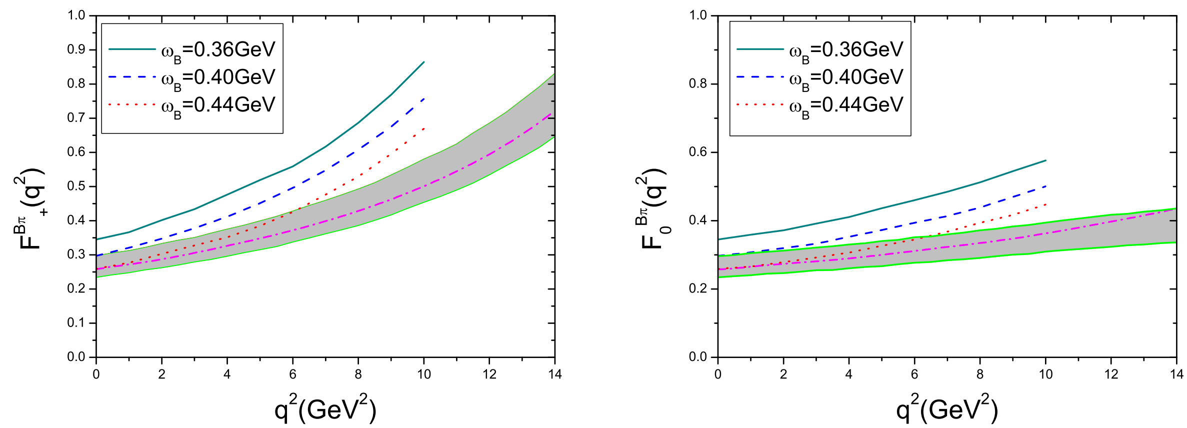


FIG. 5: Comparison of different PQCD results for the $B \to \pi$ transition form factors $F_+^{B\pi}(q^2)$ (Left) and $F_0^{B\pi}(q^2)$ (Right). The solid, dashed and dotted lines are the results obtained in Ref.[21] and are for $\omega_B = 0.36$GeV, 0.40GeV and 0.44GeV respectively. The shaded band are our present results with the upper edge for $\bar{\Lambda} = 0.50GeV$ and the lower edge for $\bar{\Lambda} = 0.60GeV$, respectively. For comparison, the dash-dot line stands from the QCD LCSR result[4, 5].

the new definition (13) for Ψ_B and $\bar{\Psi}_B$, the contribution from $\bar{\Psi}_B$ is not small ($\sim 25-40\%$) and it can not be safely neglected. Thus both Ψ_B and $\bar{\Psi}_B$ should be kept in the calculation for giving a better understanding of the B decays.

Finally, we make a comparison of the present results for $F^{B\pi}_{+,0}(q^2)$ with those obtained in Ref.[21] in Fig.(5). In Ref.[21], $\bar{\Psi}_B$ has been neglected and Ψ_B takes the form

$$\Psi_B(x, b_B) = N_B x^2 (1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{xM_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b_B^2}{2}\right], \tag{15}$$

where N_B is the normalization factor and ω_B is taken to be $(0.40 \pm 0.04)GeV$. In Fig.(5), we show their results for $\omega_B = 0.36$GeV, 0.40GeV and 0.44GeV and our present results with $\bar{\Lambda} \in (0.5GeV, 0.6GeV)$, respectively. The two results in the large recoil regions $q^2 \sim 0$ are consistent with each other, however one may observe that the fast rise in Ref.[21] has been suppressed in our present results and the slope of the present obtained form factors $F^{B\pi}_{+,\,0}(q^2)$ are more consistent with the QCD LCSR results in Ref.[4, 5]. The main reason for the differences between our present results and those in Ref.[21] is that we have used a better endpoint behavior wavefunction for Ψ_p[22]. With this new form for Ψ_p, we find that the total twist-3 contributions are in fact less than ($\sim 70\%$) the leading twist contribution

in the large recoil regions. While in Ref.[21], the twist-3 contributions are about two times bigger than that of the leading twist, especially for the bigger q^2 regions, and then the total contributions will give a fast rise in shape.

IV. DISCUSSION AND SUMMARY

In the present paper, we have examined the $B \to \pi$ transition form factor in the PQCD approach, where the transverse momentum dependence for the wavefunction, the Sudakov effects and the threshold effects are included to regulate the endpoint singularity and to derive a more reasonable result. We emphasize that the transverse momentum dependence for both the B meson and the pion is important to give a better understanding of the $B \to \pi$ transition form factor. The pionic twist-3 contributions to the $B \to \pi$ transition form factor are carefully studied with a better endpoint behavior wavefunction for Ψ_p, and Fig.(3) shows that the twist-3 contributions are of the same order of the leading twist contribution, however its values are less than that of the leading twist. This observation improves the results obtained in Refs.[12, 21], in which the asymptotic behavior for ϕ_p was used and they claimed a large twist-3 contributions to the $B \to \pi$ transition form factor, i.e. bigger than that of the leading twist. Fig.(3b) and Fig.(4) show that both Ψ_B and $\bar{\Psi}_B$ are important, no matter what definition (Eq.(8) or Eq.(13)) is chosen. Under the definition (8), the negative contribution from $\bar{\Psi}_B$ is necessary to suppress the big contribution from Ψ_B and to obtain a reasonable total contributions. While under the definition Eq.(13), the contribution from $\bar{\Psi}_B$ is power suppressed to that of Ψ_B, however it still can contribute $25-40\%$ to the total contributions. As is shown in Fig.(5), a comparison of our present results for $F^{B\pi}_{+,\,0}(q^2)$ with those in Ref.[21] shows that a better PQCD result (with its slope closes to the QCD LCSR results) can be obtained by carefully considering both the pionic twist-3 contributions and the contributions from the two wavefunctions Ψ_B and $\bar{\Psi}_B$ of the B meson.

In the literature, the values of the $B \to \pi$ transition form factors $F^{B\pi}_{+}(0)$ and $F^{B\pi}_{0}(0)$ are determined around 0.3. With $\bar{\Lambda} \in (0.50GeV, 0.60GeV)$, we obtain $F^{B\pi}_{+,0}(0) = 0.265 \pm 0.032$. This result is consistent with the extrapolated lattice QCD result $F^{B\pi}_{+,0}(0) = 0.27 \pm 0.11$[1] and the newly obtained QCD LCSR result $F^{B\pi}_{+,0}(0) = 0.258 \pm 0.031$[5]. The PQCD calculation are reliable only when the involved energy scale is hard enough. The lattice QCD calculations which presently are available only for the soft regions, i.e. $q^2 > 15GeV^2$. The

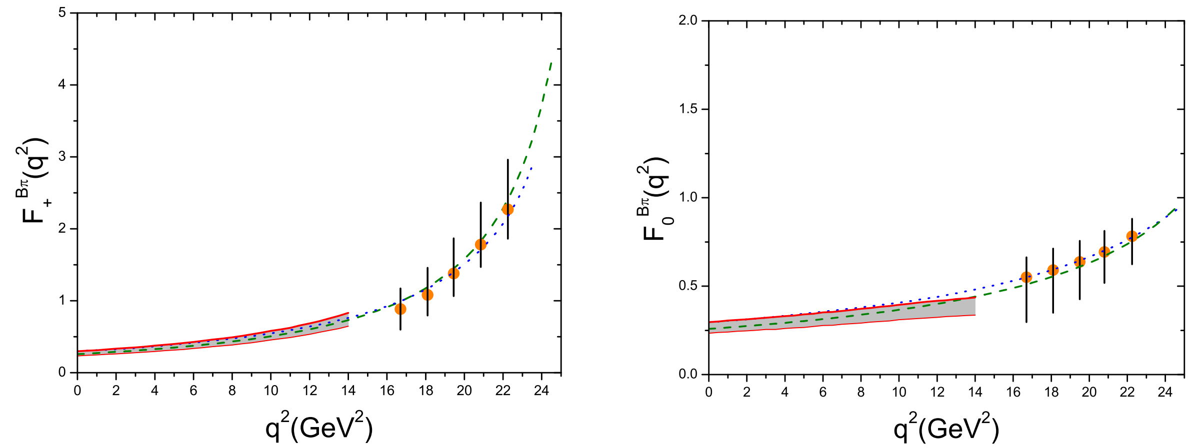


FIG. 6: PQCD results for the $B \to \pi$ form factors $F_+^{B\pi}(q^2)$ (Left) and $F_0^{B\pi}(q^2)$ (Right). The shaded band are our present results with the upper edge for $\bar{\Lambda} = 0.50GeV$ and the lower edge for $\bar{\Lambda} = 0.60GeV$, respectively. The dashed and dotted lines stand for the QCD LCSR result Eq.(16) and the fits to the lattice QCD results with errors[3], respectively.

QCD LCSR can treat both hard and soft contributions with $q^2 \precsim 18GeV^2$[4, 5] on the same footing. Therefore, the results from the PQCD approach, the lattice QCD approach and the QCD LCSRs are complementary to each other and by combining the results of those three approaches, one may obtain an understanding of the $B \to \pi$ transition form factor in the whole physical regions. The $B \to \pi$ transition form factors $F_+^{B\pi}(q^2)$ and $F_0^{B\pi}(q^2)$ derived from QCD LCSRs can be written in the following parameterization [5]:

$$F_+^{B\pi}(q^2) = \frac{r_1}{1 - q^2/m_1^2} + \frac{r_2}{1 - q^2/m_{\rm fit}^2}, \quad F_0^{B\pi}(q^2) = \frac{r_3}{1 - q^2/m_{\rm 0fit}^2}, \tag{16}$$

where r_1, r_2, r_3, m_1, $m_{\rm fit}$ and $m_{\rm 0fit}$ are fitted parameters and can be taken as[5], $r_1 = 0.744$, $r_2 = -0.486$, $r_3 = 0.258$, $m_1 = 5.32GeV$, $m_{\rm fit}^2 = 40.73GeV^2$ and $m_{\rm 0fit}^2 = 33.81GeV^2$. With the parameterization Eq.(16), the QCD LCSR results can be extrapolated up to the upper limit of q^2, i.e. $q^2 \sim 25GeV^2$, and then it can be treated as a bridge to connect both the PQCD results and the lattice QCD results. In Fig.(6), we show the results of the PQCD approach, the lattice QCD approach and the extrapolated QCD LCSR results defined in Eq.(16), respectively. Our present PQCD results with $\bar{\Lambda} \in (0.5GeV, 0.6GeV)$ are in agreement and can match with the QCD LCSR results and the lattice QCD calculations, which are shown in Fig.(6).

In summary, we have shown that the PQCD approach can be applied to calculate the $B \to \pi$ transition form factor in the large recoil regions. The twist-3 contributions are less than the leading twist contribution with a better endpoint behavior twist-3 wavefunctions and both of the two wavefunctions Ψ_B and $\bar{\Psi}_B$ of the B meson are necessary to give a deep understanding of the B decays, e.g. $B \to \pi$ transition form factor. Combining the PQCD results with the QCD LCSR and the lattice QCD calculations, the $B \to \pi$ transition form factor can be determined in the whole kinematic regions.

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APPENDIX A: FORMULAE FOR THE PION AND B MESON WAVEFUNCTIONS

To do the numerical calculations, for the pion wave functions, we take

$$\Psi_{\pi,\sigma}(x, \mathbf{k}_\perp) = A_\pi \exp\left(-\frac{m^2 + k_\perp^2}{8\beta^2 x(1-x)}\right), \tag{A1}$$

where the parameters can be determined by the normalization condition of the wave function

$$\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi(x, \mathbf{k}_\perp) = 1 \ , \tag{A2}$$

and some necessary constraints[32]. And one can construct a model wave function Ψ_p with k_T dependence in the following[22],

$$\Psi_p(x, \mathbf{k}_\perp) = (1 + B_p C_2^{1/2}(1-2x) + C_p C_4^{1/2}(1-2x)) \frac{A_p}{x(1-x)} \exp\left(-\frac{m^2 + k_\perp^2}{8\beta^2 x(1-x)}\right), \tag{A3}$$

where $C_2^{1/2}(1-2x)$ and $C_4^{1/2}(1-2x)$ are Gegenbauer polynomials and the coefficients A_p, B_p and C_p can be determined by the DA moments. In the above equations,

$$m = 290 MeV, \quad \beta = 385 MeV, \tag{A4}$$

which are derived for $\langle \mathbf{k}_\perp{}^2 \rangle \approx (356 MeV)^2$[32]. The parameters in Eq.(A3) can then be determined as,

$$A_\pi = 1.187 \times 10^{-3} MeV^{-2}, \quad A_p = 2.841 \times 10^{-4} MeV^{-2}, \quad B_p = 1.302, \quad C_p = 0.126. \tag{A5}$$

And for the B meson wave function, we take[19, 33]

$$\Psi_B^-(\xi,\mathbf{k}_\perp) = 16\pi^3\frac{2\bar{\xi}-\xi}{2\pi\bar{\xi}^2}\theta(2\bar{\xi}-\xi)\delta(k_\perp^2-M_B^2\xi(2\bar{\xi}-\xi)), \quad \text{(A6)}$$

$$\Psi_B^+(\xi,\mathbf{k}_\perp) = 16\pi^3\frac{\xi}{2\pi\bar{\xi}^2}\theta(2\bar{\xi}-\xi)\delta(k_\perp^2-M_B^2\xi(2\bar{\xi}-\xi)), \quad \text{(A7)}$$

with $\xi=\frac{l^+}{M_B}$ and $\bar{\xi}=\frac{\bar{\Lambda}}{M_B}$, where $\bar{\Lambda}$ is the effective mass of the B meson.

After doing the Fourier transformation with the formula Eq.(12), we obtain

$$\Psi_{\pi,\sigma}(x,b_\pi) = 2\pi A_\pi\int_0^{1/b_\pi}\exp\left(-\frac{m^2}{8\beta^2x(1-x)}\right)J_0(b_\pi k_\perp)k_\perp dk_\perp \quad \text{(A8)}$$

$$\Psi_p(x,b_\pi) = \frac{2\pi A_p}{x(1-x)}[1+B_pC_2^{1/2}(1-2x)+C_pC_4^{1/2}(1-2x)]\cdot \int_0^{1/b_\pi}\exp\left(-\frac{m^2}{8\beta^2x(1-x)}\right)J_0(b_\pi k_\perp)k_\perp dk_\perp \quad \text{(A9)}$$

$$\Psi_B^-(\xi,b_B) = 16\pi^3\frac{2\bar{\xi}-\xi}{2\bar{\xi}^2}\theta(2\bar{\xi}-\xi)\theta(1/b_B^2-\xi(2\bar{\xi}-\xi)M_B^2)J_0(M_Bb_B\sqrt{\xi(2\bar{\xi}-\xi)}) \quad \text{(A10)}$$

$$\Psi_B^+(\xi,b_B) = 16\pi^3\frac{\xi}{2\bar{\xi}^2}\theta(2\bar{\xi}-\xi)\theta(1/b_B^2-\xi(2\bar{\xi}-\xi)M_B^2)J_0(M_Bb_B\sqrt{\xi(2\bar{\xi}-\xi)}) \quad \text{(A11)}$$

$$\begin{aligned}\Delta(\xi,b_B) &= M_B\int_0^\xi d\xi'[\Psi_B^-(\xi',b_B)-\Psi_B^+(\xi',b_B)] \\ &= 16\pi^3M_B\theta(1/b_B^2-\xi(2\bar{\xi}-\xi)M_B^2)\int_0^\xi\frac{\bar{\xi}-\xi'}{\bar{\xi}^2}J_0\left(M_Bb_B\sqrt{\xi'(2\bar{\xi}-\xi')}\right)d\xi' \quad \text{(A12)}\end{aligned}$$

One may easily find that the effects of the upper limit $(1/b_B)$ for the B meson wave functions are quite small (numerically less than 0.1%). This is reasonable, since the B meson mass is enough heavy to give a natural separation scale.

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[1] L.D. Debbio, J.M. Flynn, L. Lellouch and J. Nieves, Phys.Lett. B**416**, 392(1998).

[2] D. Becirevic, Nucl.Phys.Proc.Suppl. **94**, 337(2001).

[3] K.C. Bowler, *etal.*, Phys.Lett. B**486**, 111(2000).

[4] T. Huang, Z.H. Li and X.Y. Wu, Phys.Rev. D**63**, 094001(2001); Z.G. Wang, M.Z. Zhou and T. Huang, Phys.Rev. D**67**, 094006(2003).

[5] P. Ball and R. Zwicky, Phys.Rev. D**71**, 014015(2005); hep-ph/0406232.

[6] A. Khodjamirian, R. Ruckl, S. Weinzierl, C.W. Winhart and O. Yakovlev, Phys.Rev. D**62**, 114002(2000).

[7] P. Ball, J. High Energy Phys. **9809**, 005(1998).

[8] M. Wirbel, B. Stech and M. Bauer, Z.Phys. C**29**, 637(1985).

[9] H.N. Li, Phys.Rev. D**52**, 3958(1995); H.N. Li and B. Melic, Eur.Phys.J. C**11**, 695(1999); C.D. Lu, K. Ukai and M.Z. Yang, Phys.Rev. D**63**, 074009(2001); M. Dahm, R. Jacob and P. Kroll, Z.Phys. C**68**, 595(1995).

[10] A. Szczepaniak, E.M. Henley and S.J. Brodsky, Phys.Lett. B**243**, 287(1990); S.J. Brodsky, SLAC-PUB-5529; S.J. Brodsky, SLAC-PUB-5917.

[11] Y.Y. Keum, H.N. Li and A.I. Sanda, Phys.Rev. D**63**, 054008(2001).

[12] Z.T. Wei and M.Z. Yang, Nucl.Phys. B**642**, 263(2002).

[13] C.D. Lu and M.Z. Yang, Eur.Phys.J. C**28**, 515(2003).

[14] Chia-HungV. Chang and H.N. Li, Phys. Rev. D**55**,5577(1997); T.W. Yeh and H.N. Li, Phys. Rev. D**56**,1615 (1997); M. Nagashima and H.N. Li, Phys. Rev. D **67**, 034001(2003).

[15] H.N. Li and H.L. Yu, Phys. Rev. Lett. **74**, 4388(1995); H.N. Li and H.L. Yu, Phys. Lett. B**353**, 301(1995); H.N. Li and H.L. Yu, Phys. Rev. D**53**, 2480(1996).

[16] G.P. Lepage and S.J. Brodsky, Phys. Rev. Lett. **43**,545(1979), Erratum-ibid. **43**, 1625(1979); S.J. Brodsky and G.P. Lepage, Phys. Lett. B**87**, 359(1979); G.P. Lepage and S.J. Brodsky, Phys. Rev. D**22**, 2157 (1980); A.Efremov and A.V. Radyushkin, Phys. Lett. B**94**, 245(1980); H.N. Li and G. Sterman, Nucl. Phys. B**381**, 129(1992).

[17] T. Huang and C.W. Luo, Commun.Theor.Phys. **22**, 473(1994); A. Khodjamirian, R. Ruckl and C.W. Winhart, Phys.Rev. D**58**, 054013(1998).

[18] G. Sterman, Phys.Lett. B**179**, 281(1986); Nucl.Phys. B**281**, 310(1987); S. Catani and L. Trentadue, Nucl.Phys. B**327**, 323(1989); Nucl.Phys. B**353**, 183(1991).

[19] T. Huang, X.G. Wu and M.Z. Zhou, hep-ph/0412225.

[20] S.D. Genon and C.T. Sachrajda, Nucl.Phys. B**625**, 239(2002).

[21] T. Kurimoto, H.N. Li and A.I. Sanda, Phys.Rev. D**65**, 014007(2002).

[22] T. Huang and X.G. Wu, Phys.Rev. D**70**, 093013(2004).

[23] T. Huang, X.H. Wu and M.Z. Zhou, Phys.Rev. D**70**, 014013(2004).

[24] M. Beneke and T. Feldmann, Nucl.Phys. B**592**, 3(2001).

[25] H.N. Li, Phys.Rev.D **66**, 094010(2002).

[26] J. Botts and G. Sterman, Nucl.Phys. B**325**, 62(1989); F.G. Cao and T. Huang, Mod.Phys.Lett. A**13**, 253(1998).

[27] T. Huang, X.G. Wu and X.H. Wu, Phys.Rev. D**70**, 053007(2004).

[28] R. Jacob and P. Kroll, Phys.Lett. B**315**, 463(1993), Erratum-ibid. B**319**, 545(1993); L.S. Kissinger and S.W. Wang, hep-ph/9403261;

[29] Z.T. Wei and M.Z. Yang, Phys.Rev. D**67**, 094013(2003).

[30] A. Pich, hep-ph/9806303; P. Ball, JHEP **9901**, 010(1999).

[31] M. Neubert, Phys.Rept. **245**, 259(1994).

[32] T. Huang, B.Q. Ma and Q.X. Shen, Phys.Rev.D**49**, 1490(1994).

[33] H. Kawamura, J. Kodaira, C.F. Qiao and K. Tanaka, Phys.Lett. B**523**, 111(2001), Erratum-ibid. B**536**, 344(2002); Nucl.Phys.Proc.Suppl. **116**, 269(2003); Mod.Phys.Lett. A**18**, 799(2003).